Appendix 2. Optimal time allocation for individual agents

We employ a Constant Elasticity of Substitution model for the various recreational activities of the lake users. Under this model, the total utility of a representative lake user based on lake j is

$$U_j = (\sum_{i=0}^{L} Z_i^r)^{1/r} \qquad \text{[A2.1]}$$

where the summation is over all possible recreational activities while based on lake j, and r is a parameter related to elasticity of substitution, defined to be1/(1-r). We will assume that the elasticity of substitution among the different activities (cf. [A2.2] below) is very high. This assumption is made primarily for analytical simplicity. It would be interesting to relax it in future work.

We will assume that the Z_i are produced by Cobb-Douglas functional forms, which are often used in economics. They display many desirable properties such as (i) diminishing returns for modelling an important class of production phenomena, (ii) constant factor shares under competitive pricing for modelling the production side (i.e. the supply side) of the economy, (iii) constant expenditure shares for modelling the demand side of the economy. The latter two properties are useful when confronting production models with the aggregated data common in economics. See Deaton and Muellbauer (1980) for extensive discussion and application of Constant Elasticity of Substitution (CES) functional forms and Cobb Douglas functional forms to economics, especially on the demand side of the economy. See Varian (1992) for more on Cobb-Douglas and CES functional forms, especially Chapter 12 for typical uses of Cobb Douglas functional forms in empirical work in economics.

The property of Cobb Douglas functional forms that interests us is the diminishing returns property, (i). It is natural in the household production approach to utility theory that we are using here to assume that production of each good, i.e. $Z_i \equiv [q\ A_i\ C_{ij}\ \tau_{ij}]^a$ that enters utility U_j in [A1.1] would would display diminishing returns to the production input $[q\ A_i\ C_{ij}\ \tau_{ij}]$. One way to capture the diminishing returns property for output y=f(x) where x is input is to assume $y=A\ x^a$ for some positive constant a, $0<a<1$. The function $y=A\ x^a$, $0<a<1$ is a Cobb Douglas form which displays diminishing returns and that is the form we are using in [A2.2] below.

Thus we assume that the the Z_i follow Cobb-Douglas production functions. Then we can write

$$U_j = \sum_{i=0}^{L} (qA_i C_{ij} \tau_{ij})^a \qquad \text{[A2.2]}$$

where the summation is over all possible activities. In this model, the summation goes from 0 (corresponding to non-angling activities) to L, where 1 through L represent the L lakes that may be fished. Non-angling activities are included in the model by expressing their benefits in the same units as fish. The expression $(q\ A_i\ C_{ij}\ \tau_{ij})^a$ represents the utility derived from fishing lake i by a representative person based on lake j. The catch is assumed proportional to catchability q times fish stock times the amount of time spent fishing $(q\ A\ \tau)$. This benefit is weighed against costs of fishing on lake i from a base on lake j, which are inversely related to the parameter C_{ij}.

The exponent a lies between 0 and 1. Thus equation A2.2 assumes that utility derived from fishing is a concave function of fishing time.

Time available for fishing and other activities is finite and given by

$$T = \sum_{i=0}^{L} \tau_i \qquad [\text{A2.3}]$$

where the summation is over all possible activities. We need the optimal allocation of time among lakes, which we will get using a LaGrange multiplier. Write

$$\Lambda = U_j + \lambda (T - \sum_{i=0}^{L} \tau_{ij}) \qquad [\text{A2.4}]$$

Then find the optima by setting the derivatives of A2.4 equal to zero and solving for λ and the τ_{ij}.

$$d\Lambda / d\tau_{ij} = a\,(q\,A_i\,C_{ij})^a\,\tau_{ij}^{a-1} - \lambda = 0 \qquad [\text{A2.5}]$$

Solve for λ in terms of time allocation to the base lake j

$$\lambda = a\,(q\,A_j\,C_{jj})^a\,\tau_{jj}^{a-1} \qquad [\text{A2.6}]$$

Then for any lake i

$$\tau_{ij} = [a\,\lambda^{-1}\,(q\,A_i\,C_{ij})^{a}]^{1/(1-a)} \qquad [\text{A2.7}]$$

Using A2.6. and A2.7 with the constraint equation A2.3 yields optimal time spent at the base lake j,

$$\tau_{jj}^{*} = T / \{1 + [(A_0 C_{jj})^b / (A_j C_{jj})^b] + \sum_{\substack{i=1 \\ i \neq j}}^{L} [(A_i C_{ij})^b / (A_j C_{jj})^b]\} \qquad [\text{A2.8}]$$

where the exponent b is $b = a / (1-a)$. In equation A2.8 for time spent at the base lake j, the summation goes from 1 to L but skips j. To account for non-fishing activities at the base lake, an additional term $(A_0\,C_{jj})^b$ is included in equation 1. A_0 represents the reward from the non-fishing activity in fish equivalents.

The optimal time allocated to fishing lake i for a person based at lake j, τ_{ij}, is

$$\tau_{ij}^{*} = \tau_{jj}^{*}\,(A_i\,C_{ij})^b / (A_j\,C_{jj})^b \qquad [\text{A2.9}]$$

Note that catchability q has cancelled out of expressions A2.8 and A2.9.